Introduction and Objectives	Motivation	Example	Path Compression	Example
O	00000	000000	00	000000

Disjoint sets CS 491 – Competitive Programming

Dr. Mattox Beckman

University of Illinois at Urbana-Champaign Department of Computer Science

Spring 2024

◆□▶ ◆□▶ ◆ □▶ ◆ □▶ ○ □ ○ ○ ○ ○

Introduction and Objectives	Motivation 00000	Example 000000	Path Compression	Example 000000

<ロト < 団 ト < 臣 ト < 臣 ト 三 の < @</p>

Objectives

- Use the up-tree data structure to model disjoint sets
- Use path compression to make this structure efficient

Introduction and Objectives	Motivation	Example	Path Compression	Example
O	•0000	000000	00	000000

What do these groups have in common?



Jacobites







Introduction and Objectives	Motivation	Example	Path Compression	Example
O	0●000	000000		000000

Disjoint Sets

Properties of disjoint sets

If a is in the same set as b, then everything in the same set as a is in the same set as b.

- Want to declare that *a* and *b* are in the same set.
- Want to check if *a* and *b* are in the same set.
- Want to ask how many elements are in the set.
- Want to check if all known elements are in the set.
- And we want to do all of these things very very quickly.
 - The datastructure for this is the up-tree.

Introduction and Objectives	Motivation	Example	Path Compression	Example
O	00000	000000		000000

The idea

Representation

- Keep a vector ds representing all your elements.
- A positive entry means this element is in the same element as the referent.
- A negative entry means this element is the root. The magnitude is the number of elements.

Initialization

► To start, initialize the vector with all -1s.

Find operation

- find(i) = i, if ds[i] < 0</pre>
- find(i) = find(ds[i]), otherwise.

Introduction and Objectives O	Motivation 00000	Example 000000	Path Compression	Example 000000

Add

Relating a and b.

- Let sa = find(a)
- Let sb = find(b)

Choices:

- sa = sb=, already in the same set.
- ds[sa] > ds[sb], then sb is the larger set.

Let ds[sb] = ds[sb] + ds[sa] and ds[sa] = sb

Symmetrically for the other case.

```
Introduction and Objectives
                    Motivation
                                 Example
                                              Path Compression
                                                              Example
                    0000
Code
   int find(int c, vi ds) {
      if (ds[c] < 0)
        return c;
      else
        return find(ds[c],ds);
   }
   int add(int c1, int c2, vi ds) {
      int sa = find(c1);
      int sb = find(c2):
      if (sa == sb)
        return sa;
      if (ds[sa] > ds[sb]) {
        ds[sb] += ds[sa]; ds[sa] = sb; return sb;
      } else {
        ds[sa] += ds[sb]; ds[sb] = sa; return sa;
                                            ł
```

Introduction and Objectives O	Motivation 00000	Example ●00000	Path Compression 00	Example 000000
Example, pt 1				
c_1		C 7		
\bigcirc				
$\begin{pmatrix} c_2 \end{pmatrix}$				
c_5				
	4			
	1 2 3 -1 -1 -1 -	4 5 6 -1 -1 -1 -	7	
next connect 2	2 and 4			

Introduction and Objectives O	Motivation 00000	Example 0€0000	Path Compression	Example 000000
Example, pt 2				
(c_1)	\frown	C 7		
\frown		X		
$\begin{pmatrix} c_2 \end{pmatrix}$)		
(C5)	c_4			
connect 2 ar				
	123	4 5 6	7	
	-1 4 -1	-2 -1 -1	-1	
next connect	3 and 6			

◆□▶ ◆昼▶ ◆臣▶ ◆臣▶ ○臣 - 今へ⊙

Introduction and Objectives O	Motivati 00000	on	E	Example 00€000		Path 00	Compression		Exa	ample 20000
Example, pt 3										
C 1				C 7)					
\frown	(c_3)									
(<i>c</i> ₅)										
	4									
connect 3 and	6									
	1 -1	2 3 4 6	3 4 5 -2	5 -1	6 -2	7 -1				
next connect 6	and 5	5				< □ ▶	(日) (三)	× ≣ >	101	୬୯୯

Introduction and Objectives O	Motiva 0000	tion O		Exa	ample 00€00			Path Compressic 00	n	Example 000000
Example, pt 4										
(c_1)		_		(C 7)				
\frown		3) ×	$\overline{}$							
$\begin{pmatrix} c_2 \end{pmatrix}$			6							
	c_4									
connect 6 au	- -									
	כ גוו	h	h	4	-	~	7			
	-1	2 4	3 6	4 -2	с -3	6 5	-1			
next connect	t 2 and	3								

Introduction and Objectives O	Motivat 00000	ion D		E> O	kample 000€0		Path Compression	E	xample
Example, pt 5 c_1 c_2 c_5 c_5		3) ×	26		(<i>c</i> ₇))			
connect 2 and	3 (wi	ll cor	nnect	· 4 te	o 5)				
	1 -1	2 4	3 6	4 5	5 -5	6 5	7 -1		
next connect 3	B and T	1					< □ > < 곌 > < 注 > <	로 ▶ - 로	୬୯୯

Introduction and Objectives O	Motiva 0000	tion O		E	xample			Path Compression	Example 000000
Example, pt 6									
c_1 c_2 c_5		3 , ,	c_6		(C ₇)				
connect 3 and	1								
	1 5	2 4	3 6	4 5	5 -6	6 5	7 -1		

Introduction and Objectives	Motivation	Example	Path Compression	Example
O	00000	000000	●○	000000

Path Compression

• One problem is that these paths can get quite long if we are unlucky.

▲□▶ ▲圖▶ ▲臣▶ ▲臣▶ ―臣 – 釣��

Solution: update find so it compresses the paths.



Introduction and Objectives	Motivation	Example	Path Compression	Example
O	00000	000000		000000

Find with Path Compression

```
int find(int c, vi ds) {
  if (ds[c] < 0)
   return c;
  else
   return ds[c] = find(ds[c],ds);
}
int add(int c1, int c2, vi ds) {
  int sa = find(c1);
  int sb = find(c2):
  if (sa == sb)
   return sa;
  if (ds[sa] > ds[sb]) {
   ds[sb] += ds[sa]; ds[sa] = sb; return sb;
  } else {
   ds[sa] += ds[sb]; ds[sb] = sa; return sa;
                                   ŀ
```

Introduction and Objectives O	Motivation 00000	Example 000000	Path Compression	Example ●00000
Example, pt 1				
c ₁		C 7		
C 5	_			
	54			
	1 2 3 -1 -1 -1	4 5 6 -1 -1 -1 -	7	
next connect 2	2 and 4			

Introduction and Objectives O	Motivation 00000	Example 000000	Path Compression	Example 0●0000
Example, pt 2				
(c_1)		C 7		
\frown		\ \		
$\begin{pmatrix} c_2 \end{pmatrix}$)		
(C5)	$\left(\begin{array}{c} c_{4} \end{array} \right)$			
connect 2 ar	d 4			
	123	4 5 6	7	
	-1 4 -1	-2 -1 -1	-1	
next connect	t 3 and 6			

◆□▶ ◆昼▶ ◆臣▶ ◆臣▶ ○臣 - 今へ⊙

Introduction and Objectives O	Motivat 00000	ion D		Ex	ample 20000		P	ath Compres	sion		Exa OC	mple ●000
Example, pt 3												
(c_1)					C 7)						
\frown		3) ×	$\overline{}$									
$\begin{pmatrix} c_2 \end{pmatrix}$			6									
(c ₅)	$\overline{}$											
	(4)											
connect 3 and	6											
	1 -1	2 4	3 6	4 -2	5 -1	6 -2	7 -1					
next connect	6 and 2	5					< □	▶◀▱▸	< Ξ > < 3	至 ▶	1	୬୯୯

Introduction and Objectives O	Motivat 00000	ion D	Ex OC	ample 20000		Pa	th Compression	Example 000●00
Example, pt 4								
(c_1)				C 7)			
		3)	\					
))					
(C5)								
	(ι_4)							
connect 6 a	and 5							
	1 -1	2 3 4 6	4 -2	5 -3	6 5	7 -1		
next conne	ct 2 and 3	3						

Introduction and Objectives O	Motivation 00000	Example 000000	Path Compression	Example 0000€0
Example, pt 5				
(c_1)	\frown	C 7		
		\backslash		
c_2)		
	c_4			
connect 2 a	and 3 (will conne	ect 4 to 5, and u	pdate 3)	

▲□▶▲□▶▲≡▶▲≡▶ ≡ のへの

next connect 3 and 1

Introduction and Objectives O	Motivation 00000		Example 000000	Path Compression	Example 00000●
Example, pt 6					
	(c ₃)	c ₆)	C 7		
connect 3 a	nd 1				
	1 2 5 5	34 55	56 -65	7 -1	